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The evolution of the universe from non-compact Kaluza–Klein theory

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Abstract. We develop a 5D mechanism, inspired by Campbell's theorem, to explain the (neutral scalar field governed) evolution of the universe from an initially inflationary expansion that has a change of phase towards a decelerated expansion and thereafter evolves towards the present day observed accelerated (quintessential) expansion.

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1 Introduction

In a cosmological context, the energy density of scalar fields has been recognized to contribute to the expansion of the universe [1], and has been proposed to explain inflation [2], as well as the presently observed accelerated expansion [3]. The observed isotropy and homogeneity of the universe do not allow for the existence of long-range electric and magnetic fields, but neutral scalar fields can have non-trivial dynamics in an expanding Friedmann–Robertson–Walker (FRW) universe. An attempt to confront the data with the predictions for a minimally coupled scalar field with an a priori unknown potential was made recently [4].

The idea that matter in 4D can be explained from a 5D Ricci-flat ($R_{AB} = 0$) Riemannian manifold is a consequence of Campbell's theorem. It says that any analytic N-dimensional Riemannian manifold can be locally embedded in a $(N+1)$-dimensional Ricci-flat manifold. This is of great importance for establishing the generality of the proposal that 4D field equations with sources can be locally embedded in 5D field equations without sources [5]. In other words, 4D matter can be induced by a 5D apparent vacuum. Campbell's theorem is closely related to Wesson's interpretation of 5D vacuum Einstein gravity [6–8]. In view of this, it would be of interest to consider the embedding of 4D cosmological solutions in 5D Ricci-flat spaces. In Wesson's theory [called space-time-matter (STM) theory], the extra dimension is not assumed to be compactified, which is a major departure from earlier multidimensional theories where the cylindricity conditions were imposed. In this theory, the original motivation for assuming the existence of a large extra dimension was to achieve the unification of matter and geometry, i.e. to obtain the properties of matter as a consequence of the extra dimensions. For example, an attempt to understand inflation [which is governed by the neutral scalar (inflaton) field], from a 5D flat Riemannian manifold was made in [11]. During inflation, the scale factor of the universe accelerates, and this acceleration is driven by the potential energy associated with the self-interactions of a scalar field. However, Campbell's theorem implies that all inflationary solutions can be generated, at least in principle, from 5D vacuum Einstein gravity [10]. But could it be possible to develop a formalism to describe the whole evolution of the universe?

The aim of this work is to develop a 5D mechanism inspired by Campbell's theorem, to explain the (neutral scalar field governed) evolution of the universe from an initially inflationary (superluminal) expansion that has a change of phase towards a decelerated (radiation and later matter dominated) expansion and thereafter evolves towards the present day observed accelerated expansion (quintessence) [12]. Although Campbell's theorem relates N-dimensional theories to vacuum $(N+1)$-dimensional theories, it does not establish a strict equivalence between them [9]. It is therefore important to determine when such theories are equivalent. Clearly, this is a more severe restriction than embeddability. Two notions of equivalence that could be considered are dynamical equivalence and geodesic equivalence. Dynamical equivalence would imply that the dynamics of vacuum N-dimensional theories is included in a vacuum $(N+1)$-dimensional theories. Alternatively, one may consider geodesic equivalence, in the

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sense of Mashhoon et al. [10]. In this case the (3+1) geodesic equation induces a $(2+1)$ geodesic equation plus a force (per unity of mass) term F^C

$$\frac{\mathrm{d}U^C}{\mathrm{d}S} + \Gamma^C_{AB} U^A U^B = F^C \,.$$

As was demonstrated in [10], for canonical metrics the requirement $F^C = 0$ holds. Hence, in such metrics it is not really an extra assumption that the motion is geodesic.

In this work we shall use the geodesic equivalence approach.

2 Formalism

To achieve our goal, we consider the canonical 5D metric [13]

$$\mathrm{d}S^2 = \epsilon \left(\psi^2 \mathrm{d}N^2 - \psi^2 \mathrm{e}^{2N} \mathrm{d}r^2 - \mathrm{d}\psi^2\right) , \tag{1}$$

where $\mathrm{d}r^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$. Here, the coordinates $(N, \mathbf{r})$ are dimensionless, the fifth coordinate ψ has spatial unities and ϵ is a dimensionless parameter that can take the values $\epsilon = 1, -1$. The metric (1) describes a flat 5D manifold in apparent vacuum ($G_{AB} = 0$). We consider a diagonal metric because we are dealing only with gravitational effects, which are the important ones in the global evolution for the universe. To describe neutral matter in a 5D geometrical vacuum (1) we can consider the Lagrangian

$$^{(5)}\mathrm{L}(\varphi, \varphi_{,A}) = -\sqrt{\left|\frac{^{(5)}g}{^{(5)}g_0}\right|}\; ^{(5)}\mathcal{L}(\varphi, \varphi_{,A}) \,, \tag{2}$$

where $|^{(5)}g| = \psi^8 \mathrm{e}^{6N}$ is the absolute value of the determinant for the 5D metric tensor with components g_{AB} (A, B take the values $0, 1, 2, 3, 4$) and $|^{(5)}g_0| = \psi_0^8 \mathrm{e}^{6N_0}$ is a constant of dimensionalization determined by $|^{(5)}g|$ evaluated at $\psi = \psi_0$ and $N = N_0$. In this work we shall consider $N_0 = 0$, so that $^{(5)}g_0 = \psi_0^8$. Here, the index "0" denotes the values at the end of inflation. Furthermore, we shall consider an action

$$I = -\int \mathrm{d}^4x \mathrm{d}\psi \sqrt{\left|\frac{^{(5)}g}{^{(5)}g_0}\right|} \left[\frac{^{(5)}R}{16\pi G} + \mathcal{L}(\varphi, \varphi_{,A})\right] ,$$

for a scalar field φ which is minimally coupled to gravity. Here, $^{(5)}R$ is the 5D Ricci scalar, which, of course, is zero for the 5D flat metric (1), and G is the gravitational constant.

Since the 5D metric (1) describes a manifold in apparent vacuum, the density Lagrangian $\mathcal{L}$ in (2) must be

$$^{(5)}\mathcal{L}(\varphi, \varphi_{,A}) = \frac{1}{2} g^{AB} \varphi_{,A} \varphi_{,B} \,, \tag{3}$$

which represents a free scalar field. In other words, we define the vacuum as a purely kinetic 5D-lagrangian on a globally 5D flat metric [in our case, the metric (1)]. In the 3D comoving frame $U^r = 0$, the geodesic dynamics $\frac{\mathrm{d}U^C}{\mathrm{d}S} = -\Gamma^C_{AB} U^A U^B$ with $g_{AB} U^A U^B = 1$ give us the velocities U^A:

$$U^\psi = -\frac{1}{\sqrt{u^2(N) - 1}} \,, \quad U^r = 0 \,, \quad U^N = \frac{u(N)}{\psi \sqrt{u^2(N) - 1}} \,,$$

which are satisfied for $S(N) = \pm |N|$. In this work we shall consider the case $S(N) = |N|$. In this representation $\frac{\mathrm{d}\psi}{\mathrm{d}N} = \psi / u(N)$, where $u(N)$ is an arbitrary function. Thus the fifth coordinate evolves as

$$\psi(N) = \psi_0 \mathrm{e}^{\int \mathrm{d}N / u(N)} \,. \tag{4}$$

Here, ψ_0 is a constant of integration that has spatial unities. From the mathematical point of view, we are taking a foliation of the 5D metric (1) with r constant. Hence, to describe the metric in physical coordinates we must make the following transformations: $t = \int \psi(N) \mathrm{d}N$, $R = r\psi$, $L = \psi(N)\ \mathrm{e}^{-\int \mathrm{d}N/u(N)}$, such that for $\psi(t) = 1/h(t)$, we obtain the 5D metric

$$\mathrm{d}S^2 = \epsilon \left(\mathrm{d}t^2 - \mathrm{e}^{2\int h(t) \mathrm{d}t} \mathrm{d}R^2 - \mathrm{d}L^2\right) , \tag{5}$$

where $L = \psi_0$ is a constant and $h(t) = \dot{b}/b$ is the effective Hubble parameter defined from the effective scale factor of the universe b. The metric (5) describes a 5D generalized FRW metric, which is 3D spatially flat [i.e., it is flat in terms of $\mathbf{R} = (X, Y, Z)$], isotropic and homogeneous. In the representation $(\mathbf{R}, t, L)$, the velocities $\hat{U}^A = \frac{\partial \hat{x}^A}{\partial x^B} U^B$ are

$$U^t = \frac{2u(t)}{\sqrt{u^2(t) - 1}} \,, \quad U^R = -\frac{2r}{\sqrt{u^2(t) - 1}} \,, \quad U^L = 0 \,, \tag{6}$$

where the old velocities U^B are U^N, $U^r = 0$ and U^ψ and the velocities $\hat{U}^B$ are constrained by the condition

$$\hat{g}_{AB} \hat{U}^A \hat{U}^B = 1 \,. \tag{7}$$

Furthermore, the function u can be written as a function of time $u(t) = -\frac{h^2}{\dot{h}}$, where the overdot represents the derivative with respect to the time. The solution $N = \mathrm{arctanh}[1/u(t)]$ corresponds to a time dependent power-law expanding universe $h(t) = p_1(t) t^{-1}$, such that the effective scale factor goes as $b \sim \mathrm{e}^{\int p_1(t)/t \mathrm{d}t}$. When $u^2(t) > 1$, the velocities U^t and U^R are real, so that the condition (7) implies that $\epsilon = 1$. [Note that the function $u(t)$ can be related to the deceleration parameter $q(t) = -\ddot{b}b/\dot{b}^2$: $u(t) = 1/[1 + q(t)]$.] In such a case the expansion of the universe is accelerated ($\ddot{b} > 0$). However, when $u^2 < 1$ the velocities U^t and U^R are imaginary and the condition (7) holds for $\epsilon = -1$. In this case the expansion of the universe is decelerated because $\ddot{b} < 0$. So, the parameter ϵ is introduced in the metric (5) to preserve the hyperbolic condition (7). Moreover, the coordinates $(\mathbf{R}, t, L)$ have physical meaning, because t is the cosmic time and $(\mathbf{R}, L)$ are spatial coordinates. Since the line element is a function of time t (i.e., $S \equiv S(t)$), the new coordinate R gives us the physical distance between galaxies separated by cosmological distances: $R(t) = r(t)/h(t)$. Note that for $r > 1$ ($r < 1$), the 3D spatial distance $R(t)$ is defined on super (sub) Hubble scales. Furthermore $b(t)$ is the effective scale factor of the universe and describes its effective 3D euclidean (spatial) volume (see below). Hence, the effective 4D metric is a spatially (3D) flat FRW one,

$$\mathrm{d}S^2 \to \mathrm{d}s^2 = \epsilon \left(\mathrm{d}t^2 - \mathrm{e}^{2\int h(t) \mathrm{d}t} \mathrm{d}R^2\right) , \tag{8}$$

and has a effective 4D scalar curvature ${}^{(4)}\mathcal{R} = 6(\dot{h} + 2h^2)$. The metric (8) has a metric tensor with components $g_{\mu\nu}$ (μ,ν take the values $0,1,2,3$). The absolute value of the determinant for this tensor is $|{}^{(4)}g| = (b/b_0)^6$. The density Lagrangian in this new frame was obtained in a previous work [14]

$$
{}^{(4)}\mathcal{L}\left[\varphi(\mathbf{R},t),\varphi_{,\mu}(\mathbf{R},t)\right] = \frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - \frac{1}{2}\left[(Rh)^2 - \frac{b_0^2}{b^2}\right]\left(\nabla_R\varphi\right)^2 , \tag{9}
$$

and the equation of motion for φ yields

$$
\ddot{\varphi} + 3h\dot{\varphi} - \frac{b_0^2}{b^2}\nabla_R^2\varphi + \left[\left(4\frac{h^3}{\dot{h}} - 3\frac{\dot{h}}{h} - 3\frac{h^5}{\dot{h}^2}\right)\dot{\varphi} + \left(\frac{b_0^2}{b^2} - h^2R^2\right)\nabla_R^2\varphi\right] = 0 . \tag{10}
$$

From (9) and (10), we obtain respectively the effective scalar 4D potential $V(\varphi)$ and its derivatives with respect to $\varphi(\mathbf{R},t)$ are

$$
V(\varphi) \equiv \frac{1}{2}\left[(Rh)^2 - \left(\frac{b_0}{b}\right)^2\right]\left(\nabla_R\varphi\right)^2 , \tag{11}
$$

$$
V'(\varphi) \equiv \left(4\frac{h^3}{\dot{h}} - 3\frac{\dot{h}}{h} - 3\frac{h^5}{\dot{h}^2}\right)\dot{\varphi} + \left(\frac{b_0^2}{b^2} - h^2R^2\right)\nabla_R^2\varphi , \tag{12}
$$

where the prime denotes the derivative with respect to φ. Equations (9) and (10) describe the dynamics of the inflaton field $\varphi(\mathbf{R},t)$ in a metric (8) with a Lagrangian

$$
{}^{(4)}\mathcal{L}[\varphi(\mathbf{R},t),\varphi_{,\mu}(\mathbf{R},t)] = -\sqrt{\left|\frac{{}^{(4)}g}{{}^{(4)}g_0}\right|}\left[\frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} + V(\varphi)\right] , \tag{13}
$$

where $\left|{}^{(4)}g_0\right| = 1$.

Furthermore, the 4D energy density ρ and the pressure p are [13]

$$
8\pi G\rho = 3h^2 , \tag{14}
$$

$$
8\pi G\mathrm{p} = -(3h^2 + 2\dot{h}) . \tag{15}
$$

Note that the function $u(t)$ can be related to the deceleration parameter $q(t) = -\ddot{b}b/\dot{b}^2$: $u(t) = 1/[1+q(t)]$. From the condition (7) we can differentiate some different stages of the universe. If $u^2(t) = \frac{4r^2(b/b_0)^2-1}{3} > 1$, we obtain that r can take the values $r > 1$ ($r < 1$) for $b/b_0 < 1$ ($b/b_0 > 1$), respectively. In this case $q < 0$, so that the expansion is accelerated. On the other hand if $u^2(t) = \frac{4r^2(b/b_0)^2-1}{3} < 1$, r can take the values $r < 1$ ($r > 1$) for $b/b_0 > 1$ ($b/b_0 < 1$), respectively. In this stage $q > 0$ and the expansion of the universe is decelerated, so that the function $u(t)$ take the values $0 < u(t) < 1$ and the velocities (6) become imaginary. Thus, the metric (8) shifts its signature from $(+,-,-,-)$ to $(-,+,+,+)$. When $u(t) = 1$ the deceleration parameter becomes zero because $\ddot{b} = 0$. At this moment the velocities (6) rotate synchronically in the complex plane, and r take the values $r = 1$ or $r < 1$, for $b/b_0 = 1$ or $b/b_0 > 1$, respectively.

On the other hand, the effective 4D energy density operator ρ is

$$
\rho = \frac{1}{2}\left[\dot{\varphi}^2 + \frac{b_0^2}{b^2}\left(\nabla\varphi\right)^2 + 2V(\varphi)\right] . \tag{16}
$$

Hence, the 4D expectation value of the Einstein equation $\left\langle H^2\right\rangle = \frac{8\pi G}{3}\left\langle\rho\right\rangle$ on the 4D FRW metric (8) will be

$$
\left\langle H^2\right\rangle = \frac{4\pi G}{3}\left\langle\dot{\varphi}^2 + \frac{b_0^2}{b^2}\left(\nabla\varphi\right)^2 + 2V(\varphi)\right\rangle , \tag{17}
$$

where G is the gravitational constant and $\left\langle H^2\right\rangle \equiv h^2 = \dot{b}^2/b^2$. Now we can give a semiclassical treatment [11] for the effective 4D quantum field $\varphi(\mathbf{R},t)$, such that $\langle\varphi\rangle = \phi_\mathrm{c}(t)$:

$$
\varphi(\mathbf{R},t) = \phi_\mathrm{c}(t) + \phi(\mathbf{R},t) . \tag{18}
$$

For consistence we take $\langle\phi\rangle = 0$ and $\langle\dot{\phi}\rangle = 0$. With this approach the classical dynamics on the background 4D FRW metric (8) is well described by the equations

$$
\ddot{\phi}_\mathrm{c} + 3\frac{\dot{b}}{b}\dot{\phi}_\mathrm{c} + V'(\phi_\mathrm{c}) = 0 , \tag{19}
$$

$$
H_\mathrm{c}^2 = \frac{8\pi G}{3}\left(\frac{\dot{\phi}_\mathrm{c}^2}{2} + V(\phi_\mathrm{c})\right) , \tag{20}
$$

where $H_\mathrm{c}^2 = \dot{a}^2/a^2$ and the prime denotes the derivative with respect to the field. In other words the scale factor a only takes into account the expansion due to the classical Hubble parameter, but the effective scale factor b takes into account both classical and quantum contributions in the energy density: $\frac{\dot{b}^2}{b^2} = \frac{8\pi G}{3}\langle\rho\rangle$. Since $\dot{\phi}_\mathrm{c} = -\frac{H_\mathrm{c}'}{4\pi G}$, from (20) we obtain the classical scalar potential $V(\phi_\mathrm{c})$ as a function of the classical Hubble parameter H_c

$$
V(\phi_\mathrm{c}) = \frac{3M_\mathrm{p}^2}{8\pi}\left[H_\mathrm{c}^2 - \frac{M_\mathrm{p}^2}{12\pi}\left(H_\mathrm{c}'\right)^2\right] ,
$$

where $M_\mathrm{p} = G^{-1/2}$ is the Planckian mass. The quantum dynamics is described by

$$
\left\langle H^2\right\rangle = H_\mathrm{c}^2 + \frac{8\pi G}{3}\left\langle\frac{\dot{\phi}^2}{2} + \frac{b_0^2}{2b^2}(\nabla\phi)^2 + \sum_{n=1}\frac{1}{n!}V^{(n)}(\phi_\mathrm{c})\phi^n\right\rangle , \tag{21}
$$

$$
\ddot{\phi} + 3\frac{\dot{b}}{b}\dot{\phi} - \frac{b_0^2}{b^2}\nabla^2\phi + \sum_{n=1}\frac{1}{n!}V^{(n+1)}(\phi_\mathrm{c})\phi^n = 0 . \tag{22}
$$

In what follows we shall make the following identification:

$$\Lambda(t) = 8\pi G \left\langle \frac{\dot{\phi}^2}{2} + \frac{b_0^2}{2b^2}(\nabla\phi)^2 + \sum_{n=1} \frac{1}{n!} V^{(n)}(\phi_c)\phi^n \right\rangle , \tag{23}$$

such that

$$\frac{\dot{b}^2}{b^2} = \frac{\dot{a}^2}{a^2} + \frac{\Lambda}{3} . \tag{24}$$

On cosmological scales, the fluctuations ϕ are small, so that it is sufficient to make a linear approximation ($n = 1$) for the fluctuations. Thus, the second term in (24) is negligible on such scales. However, the second term in (24) could be important in the ultraviolet spectrum and more exactly at Planckian scales. At these scales the modes for ϕ should be coherent and the matter inside these regions can be considered as dark. Hence, the significant contribution for the function $\Lambda(t)$ is given by

$$\Lambda(t) \simeq 8\pi G \left\langle \frac{\dot{\phi}^2}{2} + \frac{b_0^2}{2b^2}(\nabla\phi)^2 + \sum_{n=1} \frac{1}{n!} V^{(n)}(\phi_c)\phi^n \right\rangle \Bigg|_{\text{Planck}} . \tag{25}$$

In this sense, we could make the identification for Λ as a cosmological parameter which only takes into account the "coherent quantum modes" (or dark matter) contribution for the expectation value of energy density: $\langle \rho_\Lambda \rangle = \Lambda/(8\pi G)$. For simplicity, in the following we shall consider Λ as a constant.

Once having done the linear approximation ($n = 1$) for the semiclassical treatment (18), we can make the identification of the squared mass for the inflaton field $m^2 = V''(\phi_c)$ [16]. Hence, after we make a linear expansion for $V'(\varphi)$ in (12), we obtain

$$V'(\phi_c) \equiv \left(4\frac{h^3}{\dot{h}} - 3\frac{\dot{h}}{h} - 3\frac{h^5}{\dot{h}^2} \right) \dot{\phi}_c , \tag{26}$$

$$m^2\phi \equiv \left(4\frac{h^3}{\dot{h}} - 3\frac{\dot{h}}{h} - 3\frac{h^5}{\dot{h}^2} \right) \frac{\partial\phi}{\partial t} + \left(\frac{b_0^2}{b^2} - h^2R^2 \right) \nabla_R^2\phi . \tag{27}$$

Taking into account the expressions (19) with (26) and (22) with (27), we obtain the dynamics for ϕ_c and ϕ. Hence, the equations $\ddot{\phi}_c + 3h\dot{\phi} + V'(\phi_c) = 0$ and $\ddot{\phi} + 3h\dot{\phi} - (b/b_0)^2\nabla_R^2\phi + V''(\phi_c)\phi = 0$ now take the form [14]

$$\ddot{\phi}_c + [3h + f(t)]\,\dot{\phi}_c = 0 , \tag{28}$$

$$\ddot{\phi} + [3h(t) + f(t)]\,\dot{\phi} - h^2R^2\nabla_R^2\phi = 0 , \tag{29}$$

where

$$f(t) = \left(4\frac{h^3}{\dot{h}} - 3\frac{\dot{h}}{h} - 3\frac{h^5}{\dot{h}^2} \right) . \tag{30}$$

3 An example

To illustrate the formalism we consider a time dependent power expansion $p(t) = 2/3 + At^{-2} - Bt^{-1}$, such that the classical Hubble parameter is given by $H_c(t) = p(t)/t$ and (A, B) are constants. The effective power $p_1(t)$ for the effective Hubble parameter $h(t)$ will be

$$p_1(t) = \sqrt{(2/3 + At^{-2} - Bt^{-1})^2 + \Lambda/3t^2},$$

because $h^2 = H_c^2 + \Lambda/3$. In what follows we shall consider the universe as spatially flat. This implies that the total density parameter will be $\Omega_T = \Omega_r + \Omega_m + \Omega_\Lambda = 1$, for a critical energy density given by $\rho_c = \frac{3}{8\pi G}h^2$, such that

$$\Omega_r + \Omega_m = \frac{H_c^2}{h^2} , \qquad \Omega_\Lambda = \frac{\Lambda}{3h^2} , \tag{31}$$

where Ω_r, Ω_m and Ω_Λ are respectively the contributions for radiation, matter and Λ. In our case, because we consider $\Omega_T = 1$, this implies that

$$p_1^2(t) = p^2(t) + \frac{1}{3}\Lambda t^2 , \tag{32}$$

where $t > 0$ is the cosmic time. We define $b/b_0 = e^N$, such that $b_0 \equiv b(t = t_0)$, where we shall consider t_0 as the time when inflation ends (i.e., the time for which $\ddot{b} = 0$). Thus N will be grater than zero only for times larger than t_0, but negative for $t < t_0$ (i.e., during the previous inflationary phase). This means that the parameter N gives us the number of e-folds with respect to the scale factor at the end of inflation: b_0. Once we have defined the scale for N, we can see the evolution for the function $u(t)$. During inflation $\ddot{b} > 0$, so that $u(t) > 1$ and $\epsilon = 1$. In such an epoch $q < 0$ (i.e., the universe is accelerated) and $b/b_0 = e^N < 1$, because $N < 0$. In such a phase the parameter r obeys $r \gg 1$. This means that cosmological scales include regions very much larger than the Hubble horizon [see the metric (8)].

At the end of inflation $u(t)$ take values close to (but larger than) unity. At $t = t_0$ $\ddot{b} = q = 0$, the function $u(t_0) = 1$, so that the global hyperbolic geometry condition $\hat{g}_{AB}\hat{U}^A\hat{U}^B = 1$ is not well defined [see (6)]. However, the line element (8) is well defined. At this moment the universe suffers a change of phase from an accelerated to a decelerated expansion and $r = 1$, because $b(t = t_0) = b_0$.

During the second phase (i.e., decelerated expansion) the universe is governed by radiation and later by matter. The function $u^2(t)$ is smaller than unity (but $u^2 > 0$), so that r takes values $\frac{1}{2}e^{-N} = \frac{1}{2}b_0/b < r < 1$, for $N > 0$. This means that, during this phase, the metric (8) describes the universe on scales smaller than the Hubble radius: $r/h < 1/h$. The interesting thing here is that the velocities (6) become purely imaginary and the signature of the 4D effective metric (8) changes synchronically (with respect to the signature during the inflationary phase): $(+,-,-,-) \to (-,+,+,+)$; that is, ϵ jumps from 1 to -1 to preserve the global geometry in (7). In this sense we can say that the 4D effective metric (8) is "dynamical". Note that this possibility was first considered by Davidson and Owen in [17]. Fig. 1 shows the evolution of the

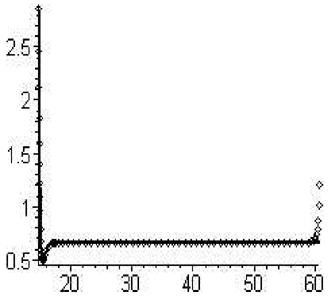

Fig. 1. Evolution of $p_1[x(t)]$ (dashed line) and $p[x(t)]$ (continuous line) as a function of $x(t) = \log_{10}(t)$, for $A = 1.5\,10^{30}\,G^1$, $B = 10^{15}\,G^{1/2}$

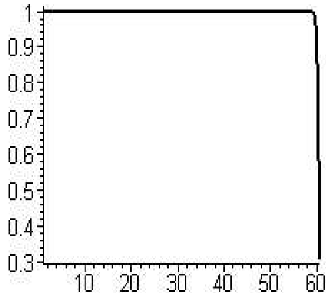

Fig. 2. Evolution of $(\Omega_{\rm m} + \Omega_{\rm r})[x(t)]$ as a function of $x(t) = \log_{10}(t)$, for $A = 1.5\,10^{30}\,G^1$, $B = 10^{15}\,G^{1/2}$

powers $p_1[x(t)]$ (dashed line) and $p[x(t)]$ (continuous line) as a function of $x(t) = \log_{10}(t)$ for $A = 1.5\,10^{30}\,G^1$ and $B = 10^{15}\,G^{1/2}$. Numerical calculations give us the time for which $\ddot{b} = q = 0$ at the end of inflation: $x(t_0) \simeq 14.778$. At this moment $N(t_0) = 0$, but after this it becomes positive. Note that for $x(t) < 60.22$ both curves are very similar, but for $x(t) > x(t_*)$ (with $x(t_*) \simeq 60.22$), p_1 increases very rapidly but not p, which remains almost constant with a value close to $p \simeq 2/3$. The difference between both curves is due to the presence of a non-zero "cosmological constant" (Λ), which was given the value of $\Lambda = 1.5\,10^{-121}\,G^{-1}$. [At the moment consensus has emerged on the experimental value of the cosmological constant [18,19]. It is of the order of magnitude of the matter energy density: $\rho_\Lambda \sim (2\text{–}3)\rho_{\rm m}$. The Wilkinson Microwave Anisotropy Probe (WMAP) data suggest that the universe is very nearly spatially flat, with a density parameter $\Omega_{\rm T} = 1.02 \pm 0.02$ [20].] In other words, at $t_* \simeq 1.66\,10^{60}\,G^{1/2}$ the deceleration parameter becomes zero and later negative. At this moment, the universe changes from a decelerated to an accelerated phase and ϵ jumps from -1 to 1 because $u(t)$ evolves from $u(t < t_*) < 1$ (decelerated expansion) to $u(t > t_*) > 1$ (accelerated expansion). It should be when the universe was nearly $0.4\,10^{10}$ years old. The present day age of the universe was considered as $x(t) = 60.653\,G^{1/2}$ (i.e., $1.5\,10^{10}$ years old). Note that $\Omega_{\rm r} + \Omega_{\rm m}$ decreases for late times [see Fig. 2], so that its present day value should be $(\Omega_{\rm r} + \Omega_{\rm m})[x(t = 60.653\,G^{1/2})] \simeq 0.32$. Thus, the present day value for the vacuum density parameter $\Omega_\Lambda = 1 - (\Omega_{\rm r} + \Omega_{\rm m})$ should be $\Omega_\Lambda[x(t = 60.653\,G^{1/2})] \simeq 0.68$. With these parameter values we obtain the present day deceleration parameter: $q[x(t = 60.653\,G^{1/2})] \simeq -0.747$, so that the present day cosmological parameter should be $\omega[x(t = 60.653\,G^{1/2})] \simeq -0.831$. Note that all these results are in very good agreement with observation [15,20].

4 Final comments

The possibility that our universe is embedded in a higher dimensional space has generated a great deal of active interest. In brane-world and STM theories the usual constraint on Kaluza–Klein models, namely the cylinder condition, is relaxed so the extra dimensions are not restricted to be compact. Although these theories have different physical motivations for the introduction of a large dimension, they share the same working scenario, and lead to the same dynamics in 4D [21]. In this work we have studied a model for the evolution of the universe which is globally described by a single scalar field from a 5D apparent vacuum. Such a vacuum is described by the diagonal metric (1) and a purely kinetic Lagrangian. The 5D formalism here developed could be extended to other particular frames or quantum fields. Moreover, the evolution of the universe could be examined taking into account also electromagnetism by introducing off-diagonal terms in the metric [22], which should be relevant to the study of 3D spatial anisotropies in the universe on astrophysical scales. However, all these issues go beyond the scope of this work.

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